THE INFLUENCE OF CURRENTS ON WAVE HEIGHT ESTIMATES FROM PRESSURE AND VELOCITY MEASUREMENTS

C.F. Neves[1] and L.A.M. Endres[2]

Paper topic: Laboratory and field observations and techniques

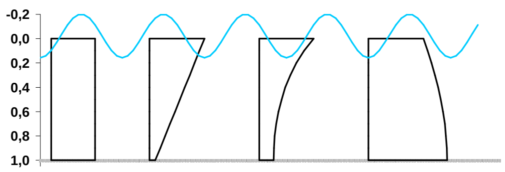
1. Introduction

Some wave measurement devices estimate wave heights based on pressure and velocity measurements, thus requiring transfer functions which transform the recorded quantity on surface elevation position. Such transfer functions are built in the equipment electronics following hypothetical fixed conditions: linear theory, just one direction, no currents. These conditions very often are not met in the field, though. Tidal currents are always present, bimodal spectra are common to many coastal locations, and linear theory is not adequate on extreme wave conditions.

Older models of pressure wave gauges did not measure currents; newer devices measure simultaneously either pressure and local current at instrument level, or pressure and current profile. It is not clear, from instrument description, whether and how the current is introduced on the computation of wave height. This investigation was then motivated by the need to quantifying, on a theoretical basis, the error on wave height estimates from pressure records, if the effect of the currents on the waves were disregarded. One could also extend this investigation to nonlinearities and vorticity effects (e.g. Peregrine 1976).

2. Theoretical background

Numerical solutions for the Stream Function may be found for the following cases (Dalrymple, 1974; Dalrymple and Cox, 1976):



CasoI : $-\psi_{xx} - \psi_{zz} = 0$

CasoII : $-\psi_{xx} - \psi_{zz} = \Omega_0$

CasoIII : $-\psi_{xx} - \psi_{zz} = -\gamma^2\psi$

CasoIV : $-\psi_{xx} - \psi_{zz} = \gamma^2\psi$

Figure 1: Current velocity vertical profile for Cases I to IV (left to right).

This numerical theory can be extended to other vorticity profiles, as reviewed by Neves (1987).

Linear analytical solutions exist for waves propagating on a uniform current and on a constant vorticity current. Linear dispersion relations are given, respectively, by equations (1) and (2), which are shown in Figure 2.

$$(\omega - kU_0)^2 = gk \tanh kh \quad \text{......(1)}$$

$$(\omega - kU_s)^2 = \{gk - \Omega_0[\omega - kU_s]\}\tanh kh \quad \text{......(2)}$$

For the irrotational case (I), the pressure response function, K_p, is given by equation (3) regardless of the presence of a (uniform) current. It turns out, though, that the wave number is significantly different if, depending on the value of U_0 which is introduced in equation (1).

For a current with constant vorticity, Case II, the expression for the pressure response function (equation 4) is rather more complex than that for the irrotational case. This shows how much the presence of vorticity changes the wave internal dynamics.

[1] Federal University of Rio de Janeiro, neves@peno.coppe.ufrj.br
[2] Federal University of Rio Grande do Sul, endres@ufrgs.br

$$K_p(z,\omega)=\frac{\cosh[k(z+h)]}{\cosh kh} \quad \text{...........................(3)}$$

$$K_{2p}(z,\omega)=\left\{1-\frac{(\omega-kU_s)^2}{gk\tanh kh}\right\}\frac{\operatorname{senh}[k(z+h)]}{\operatorname{senh} kh}+ \\ +\frac{\omega-k[U_0+\Omega_0(z+h)]}{\omega-kU_s+\Omega_0\tanh kh}\frac{\cosh[k(z+h)]}{\cosh kh} \quad \text{.......... (4)}$$

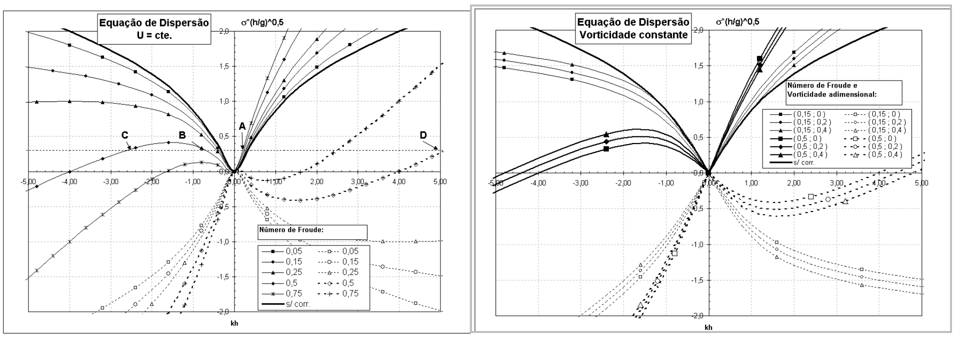


Figure 2: Dispersion relation for Cases I (left) and II (right). Points A, B, C, D indicate admissible solutions.

3. Numerical results

For given values of wave height, wave period relative to the bottom (or apparent period), water depth and (uniform) current velocity, the Stream Wave Function Theory (Dalrymple, 1974) computes the wave length, orbital velocities and acceleration, and pressure at any point within the fluid domain. Knowing the apparent period and the mean water depth, the linear, no current, pressure response function is applied to the non linear values of the pressure along the wave length, and a false free surface elevation is thus obtained. The false wave height is then compared to the actual wave height which was used as an input for the numerical model.

Numerical experiments were conducted for all combinations of: water depth (h) equal to 5, 10 and 20m; wave heights (H) of 1, 2, 3, and 4m; wave periods (T) of 6, 7, 8, 9, and 10 s; current velocity (U_0) equal to 0, 0.5, 1.0, 1.5, and 2.0 m/s, either following or running against the wave.

Relative errors in wave length range from -17% to 31%. In wave height (or pressure), though, errors may range from -50% (following current) to more than 300% (opposing current).

4. Experimental results

One of the most challenging tasks for the studies of waves superimposed on currents is the correct assessment of the velocity (or vorticity) profile. Several experimental studies have been conducted in the flumes of the Institute of Hydraulic Research at the Federal University of Rio Grande do Sul. These studies are reviewed in order to obtain typical velocity profiles, as a first step to further investigation on wave and current interaction.

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