A WAVE PROPAGATION MODEL OVER A RAPIDLY VARYING BOTTOM TOPOGRAPHY

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Paper topic: Coastal waves, currents, tides and storm surges

1. Introduction

The successful application of a coastal engineering project depends greatly on a reliable prediction of the existing wave field at the location of the project. This is usually achieved by running MSE, Boussinesq or other hyperbolic type mathematical models. In most cases, these models rely on the assumption of a mildly sloping sea bed over the coastal area of application. However, this is not true whenever underwater shoals or trenches (or other anomalies) are present in the sea bed or coastal projects of an abrupt geometry are planned featuring high side slopes as, for example, artificial reefs, navigation channels, sand pits etc. Therefore, it is necessary that conventional wave propagation models are modified in this respect by adding extra terms in their equations (Suh et al, 1997; Madsen et al, 2006) to account for bottom slope and curvature with the view of properly simulating the complex wave transformation taking place over physical or technical sea bottom obstacles. Such a model is herein proposed for the solution of this important problem incorporating also the effects of the generated evanescent waves.

2. Solution Model

By applying the principles of mass and momentum conservation over a vertical water column of infinitesimall dimensions, the following depth-averaged continuity and momentum equations are derived for an irrotational fluid :

$$\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}\left[U.(h+\eta)\right]=0 \qquad (1)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+W\frac{\partial W}{\partial x}=-\frac{1}{h}\frac{\partial}{\partial x}\left(C^2.\eta\right)+g.\frac{\eta}{h}\frac{1}{\cosh(k.h)}\frac{\partial h}{\partial x} \qquad (2)$$

where η =elevation, h =mean depth, U,W =mean depth horizontal and vertical velocity, $C^2=\frac{g}{k}\tan h(k.h)=$ wave celerity. The equations are integrated in space-time by an appropriate Method of Characteristics (MOC) numerical technique. The velocity W is calculated by integrating the vertical momentum equation.

3. Model Validation and Verification

The proposed numerical model is extensively tested and validated by comparing the calculated by the model reflection and transmission coefficients against a number of well known benchmark tests and available analytical and numerical solutions as follows: 1. Reflection from a plane shelf (ie Booij's experiment) 2.Reflection from a symmetric trench with sloped transitions and also from a Gaussian shaped trench 3.Bragg scattering from a singly sinusoidal sea bed 4. Bragg scattering from a doubly sinusoidal sea bed of large amplitude undulations 5. Normal and angled reflection from an underwater step or trench. Ongoing research is focusing on two more important issues: (a)transmission of random waves over a rippled sea bed patch as examined by Suh et al (1997) and (b)nonlinear transmission over an undulated sea bottom as examined by Liu and Yue (1998).

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4. Results and Conclusions

In test case 1 above, an excellent agreement is achieved between the calculated reflection coefficients and those of the improved modeling method presented by Suh et al (1998) and the same also holds true for test case 2 compared to the results of Bender and Dean (2003). These two tests verify that our method can simulate wave propagation over bottom slopes approaching almost the vertical.

Further, extremely good comparison is achieved in the test case 3 against the classical laboratory experiments of Davies and Heathershaw (1984) verifying the recent findings (Madsen et al , 2006) that, in fact, the principal harmonic of reflection occurs at a frequency slightly lower from the theoretically predicted. In test case 4 above, the extremely good comparison with both the experimental but mainly with the numerical results of Guazzelli et al (1992) is depicted in Fig. 1 below. Similar is the outcome in the case 5 where it is observed that the presence of evanescent modes introduces certain modifications of the analytical results in the short wave region. Based on these comparison tests, it is generally concluded that the proposed model is, indeed, a powerful tool in predicting wave behavior over a rapidly varying sea bed topography to be safely applied in coastal engineering projects.

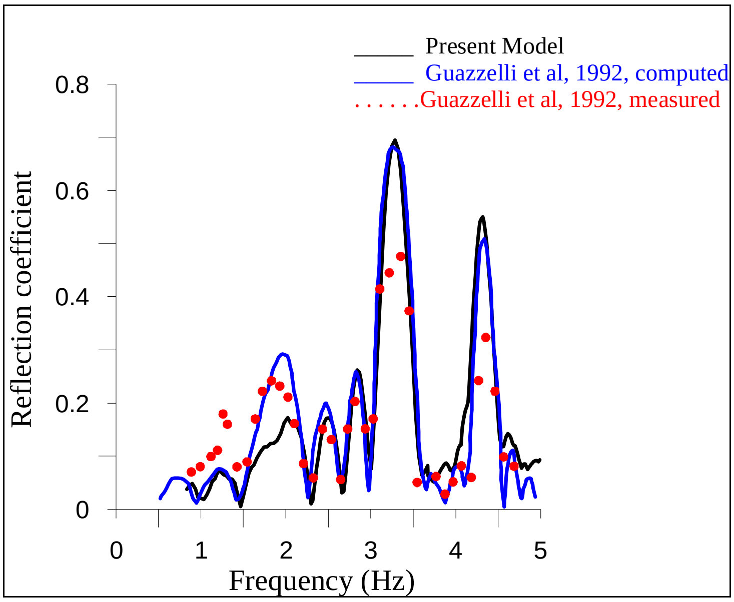

Figure 1. Bragg reflection from a doubly sinusoidal ripple bed

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